Unified treatment of contact, friction and shock-propagation in rigid body animation

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Figure 1: We model non-smooth friction (bunnies, left) and shock propagation (wall impact, right) in a single method.

ABSTRACT

We present a rigid body animation technique which prevents solids from interpenetrating, dissipates energy through friction, and propagates shocks through contacts. We employ the Alternating Direction Method of Multipliers (ADMM) to couple non-smooth Coulomb friction with impact propagation, allowing efficient and accurate non-smooth dynamics along with a correct transmission of impacts through assemblies of rigid bodies. We further extend our method to model adhesion, dynamic friction and lubricated contact.

KEYWORDS

rigid body mechanics, non-smooth dynamics, friction, adhesion

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1 RELATED WORK

Although the literature on rigid body contact and friction is rich and diverse, no single method simulates non-smooth static friction while preserving symmetries and propagating impacts [Smith et al. 2012]. For example, Gauss–Seidel-like solvers, which iterate over all contacts [Erleben 2017; Müller et al. 2020] Traditional non-smooth optimization based techniques [Acary et al. 2011; Nguyen and Brogliato 2018; Silcowitz et al. 2009] may be modified to include shock propagation using impact laws, but they are often expensive. Methods based on implicit integration of penalty methods [Ferguson et al. 2021; Lan et al. 2022; Macklin et al. 2020] are more efficient but require a smoothed *approximation* of Coulomb friction.

Our method *(1)* retains the exact non-smooth friction, *(2)* includes shock propagation, and *(3)* models extended contact laws like adhesion, for enhancing animations of rigid bodies assemblies.

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2 OUR METHOD

ALGORITHM 1: Our solver for friction and shock propagation.

Detect and merge contacts (Sec. 2.3);
for $k \leftarrow 1, 2, \ldots, N_{max}^{GS}$ **do**
  Solve the normal forces for $\mathbf{r}_N$ (Sec. 2.1, 2.2);
  Break if the error $\mathbf{f}_T^{AC}(\mathbf{u}_N; \mathbf{r}_N) \leq \varepsilon_{AC}$ (Sec 2.3);
  Solve the static friction for $\mathbf{r}_{T|S}$ (Sec. 2.1);
  (Optional) Solve the dynamic friction part for $\mathbf{r}_{T|D}$ (Sec. 2.4);

2.1 Dynamic equation with frictional contact

Following Moreau [1988], adding contact forces $\mathbf{r}$ of the Signorini–Coulomb frictional contact law to a linear dynamic equation $\mathbf{Mv} = \mathbf{f}$ (explicit Euler in our case) yields a system of the form

$$\begin{cases} \mathbf{Mv} = \mathbf{f} + \mathbf{H}^\top \mathbf{r} \\ \mathbf{u} = \mathbf{Hv}, \quad (\mathbf{u}, \mathbf{r}) \in C_\mu \end{cases} \tag{1}$$

Similarly to Tasora et al. [2021], we use an *ADMM solver with Nesterov acceleration* to solve the constrained dynamics up to a tolerance $\varepsilon_A = 10^{-4}$. However, we split the system according to the normal and tangential components of the contacts, yielding two systems to be solved alternatively until convergence [Panagiotopoulos 1975]. Doing so allows shock propagation to be introduced when solving for the normal components.

2.2 Incorporating shock propagation

Following Smith et al. [2012], we solve only for approaching contacts. We also use their strategy to handle *inelastic collapse*. We limit the number of shock propagation iterations by allowing each contact to appear at most $N_{max}^{SP} = 50$ times and solve one global system gathering all the contacts at the end.

2.3 Additional implementation details

We merge the contacts of each contiguous contact area with K-means clustering ($K = 3$). This technique increases the performance of the resolution at the cost of a simplified contact surface. For the termination, we end Alg. 1 in $N_{max}^{GS} = 10$ iterations, or stop early via a physical-based error metric with Alart and Curnier [1991]'s function $\mathbf{f}^{AC}(\mathbf{u}, \mathbf{r}) \leq \varepsilon_{AC}(= 10^{-2})$. We use only the tangential part of $\mathbf{f}^{AC}$, as the normal component is invalid due to shock propagation.

2.4 Contact law extension

We add adhesion by simply shifting the constraint on $\mathbf{r}_N$ to allow negative values. To incorporate velocity-dependent friction, such as *dynamic* friction or lubricated contacts, we compute a correction term $\mathbf{r}_{T|D} = m_D(\gamma \left\|\mathbf{u}_T(\mathbf{r}_{T|S}, \mathbf{r}_{T|D})\right\|)$ using backward Euler. The function m_D models the *smoothed* discrepancy w.r.t. the static regime $\mathbf{r}_{T|D} \approx \mathbf{r}_T - \mathbf{r}_{T|S}$ and γ a user-specified falloff parameter.

3 RESULTS

We implemented our algorithm in C++ using *OpenMP* for simple loop parallelization. All examples, depicted in the accompanying video, are run on a desktop computer with an AMD Ryzen 7 5800X 8-core processor and 64 GB of RAM.

Our method robustly handles contacts, even in the presence of large mass ratio between the elements, or a low number of ADMM iterations. The average computation time for a timestep varies between a few microseconds to $\sim 2.4s$ for our scenes composed of hundreds of rigid bodies. Shock propagation adds visual realism and reduces the number of persisting contacts but at the cost of a large overhead. For instance, for the **Wall**, shock propagation reduces the number of contacts by 33%, but the computation time increases from $1.5s$ to $2.4s$ per timestep. The non-smooth static friction is accurately captured, and our extension convincingly emulates the effects of dynamic friction and lubricated contacts.

Similarly to other local-global methods, monitoring the error metric described in Sec. 2.3 shows a fast decrease over the first iterations before a slowdown. Our contact merging scheme helps accelerate the convergence. Disabling it in the **Chain** for example yields a simulation where the chain wiggles spontaneously, requiring an increase in ADMM precision to $\varepsilon_A = 10^{-6}$.

4 LIMITATIONS

Our approach is based on the Delassus operator $\mathbf{W} = \mathbf{H}\mathbf{M}^{-1}\mathbf{H}^{\intercal}$ which robustly couples the local problems to the dynamic equation. However, this operator becomes denser with elastic bodies and can become ill-conditioned when contacts outnumber degrees of freedom. Ergo, methods to simplify this operator (e.g. [Otaduy et al. 2009; Zeng et al. 2022]), and to preserve a good convergence rate even when contact points are overabundant [Alart 2014] are potential and interesting future works.

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