Letter

Direct and efficient detection of quantum superposition

Daniel Kun ,[1,*] Teodor Strömberg ,[1,2] Michele Spagnolo,[1] Borivoje Dakić ,[1]
Lee A. Rozema,[1,†] and Philip Walther [1,3]

[1] *University of Vienna, Faculty of Physics, Vienna Center for Quantum Science and Technology (VCQ) and Research Network Quantum Aspects of Space Time (TURIS), Boltzmanngasse 5, 1090 Vienna, Austria*
[2] *Institute of Science and Technology Austria, Klosterneuburg, Austria*
[3] *Christian Doppler Laboratory for Photonic Quantum Computer, Faculty of Physics, University of Vienna, 1090 Vienna, Austria*

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One of the most striking quantum phenomena is superposition, where one particle simultaneously inhabits different states. Most methods to verify coherent superposition are indirect, in that they require the distinct states to be recombined. Here, we adapt an XOR game, in which a "test" photon is placed in a superposition of two orthogonal spatial modes, and each mode is sent to separated parties who perform local measurements on their modes without reinterfering the original modes. We show that by using a second identical "measurement" photon the parties are nonetheless able to verify if the test photon was placed in coherent superposition of the two spatial modes. We then turn this game into a resource-efficient verification scheme, obtaining a confidence that the particle is superposed which approaches unity exponentially fast. We demonstrate our scheme using a single photon, obtaining a 99% confidence that the particle is superposed with only 37 copies. Our work shows the utility of XOR games to verify quantum resources, allowing us to efficiently detect quantum superposition without reinterfering the superposed modes.

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Introduction. Superposition is a phenomenon at the heart of quantum theory [1–6] and is an essential resource for all quantum technologies, with several protocols explicitly relying on superposed single particles [7–11]. It is also of fundamental interest, where, for example, superposition is demonstrated using ever larger quantum states [12–14] to probe the limits of quantum theory. In both cases, superposition must be characterized. Conventionally, this is done *indirectly* by recombining the superposed states, varying their relative phase, and observing interference fringes to reveal superposition. However, the current effort to deploy quantum technologies [15,16] has led to a growing need to verify quantum resources in a distributed scenario. In this case, quantum resources are shared between spatially separated parties and it is not always possible to recombine and interfere the superposed states. To this end, one can ask if the superposition of a *single particle* can be verified *directly*, without interfering the two modes in which the particle is superposed. While this can be accomplished through the experimental violation of Bell's inequality [17–19] using a single photon in a superposition of two spatial modes [20–25], these methods require complex homodyne measurements. Moreover, when it comes to verifying entanglement, violating Bell's inequality is often inconvenient and replaced with device-dependent techniques, such as quantum state tomography or entanglement witnesses. By taking advantage of the interference of indistinguishable photons, we present a protocol that fulfills the same role, allowing for a direct, device-dependent verification of quantum superposition at a distance.

In this Letter, we adapt an XOR game recently proposed by Del Santo *et al.* [26] explicitly designed to *directly* detect the presence of coherent superposition. In their work, a classical bound on the probability of winning this game is derived by assuming that the information carrier is a classical particle, localized in one of two spatial modes. Exceeding this winning probability thus directly reveals the presence of a *delocalized* (i.e., spatially superposed) particle. In this protocol, we make use of an ancillary identical, superposed particle, which gives rise to two-photon interference and allows us to exceed the classical winning probability. To offset the additional resource costs, we then adapt methods from recent work on efficient entanglement detection [27,28] to our task. In particular, we measure a small, fixed number of particles and ask how likely it would be for classical particles to reproduce our observed outcomes. Doing so yields a confidence that the particle is superposed, which approaches unity exponentially fast with the number of measured particles. We experimentally demonstrate this protocol with minimal resources by placing a single test photon in a spatial superposition of two modes and verify that this test photon is superposed across these modes using only local measurements and one additional superposed ancilla photon.

Our experiment makes use of two single-photon states, independently placed in spatial superpositions. The two parts of the first spatial superposition state then interact with the

*Contact author: daniel.kun@univie.ac.at
†Contact author: lee.rozema@univie.ac.at

respective parts of the second superposition state on a beam splitter, forming a delocalized interferometer. Similar to a standard interferometer, which encodes phase information in the relative intensity of the two output spatial modes, this interferometer encodes the phase of the first superposition state in two-photon spatial correlations after the beam splitters. This allows us to observe this phase without reinterfering the spatially separated modes of the test particle. To quantify this, we adapt the proposal from Ref. [26] and formulate a two-player XOR game [29] that can be played using our interferometer. Such interferometers have been proposed [30,31] and recently demonstrated [32] to extend the baseline in long-baseline interferometry. Similar interferometers have recently also been used to teleport qubits encoded in the Fock basis [33], and have been realized on a chip [34] and with time bins [35].

Our game consists of a Referee challenging the two players, Alice and Bob, to guess the XOR value of two randomly chosen bits. As illustrated in Fig. 2, the Referee acts on a single *test photon* (T) that is sent to Alice or Bob and encodes his two bits by acting on the two respective spatial modes. Alice and Bob can locally measure the photon sent by the Referee, and are allowed to exchange classical information. They additionally share a second *ancillary measurement photon* (M), which is prepared in a spatial superposition between their two laboratories and acts as a measurement resource. In Ref. [26] it was shown that if the test particle is classical, i.e., in a statistical mixture of the two spatial modes, then Alice and Bob can do no better than to randomly guess the XOR value. This is because a classical particle can only contain information about a single bit, since it definitively travels along one of the two paths. However, if the particle is in a coherent superposition of both paths, Alice and Bob can perform a joint measurement on the test particle and their shared resource state, which allows them to correctly guess the XOR value more often.

The protocol. The goal of our protocol is for two separated users, Alice and Bob, to verify the spatial spatial superposition of a *test photon*. This superposition is prepared as shown in the orange shaded region of Fig. 1. A test photon is sent to a 50/50 beam splitter placing it in superposition of two modes, A_T and B_T, which represent the spatial modes of the test photon that are sent to Alice and Bob, respectively. More precisely, the initial state is $|1\rangle_T = \hat{a}_T^\dagger$, which the beam splitter then transforms into $\frac{1}{\sqrt{2}}(\hat{a}_{A_T}^\dagger + \hat{a}_{B_T}^\dagger)|0\rangle$. Thus the spatial superposition that Alice and Bob wish to verify is $|\psi_T\rangle \equiv \frac{1}{\sqrt{2}}(|1,0\rangle_{A_T,B_T} + |0,1\rangle_{A_T,B_T})$.

The Referee then performs *interventions* on these two paths, x in mode A_T and y in mode B_T, where $x, y \in \{0, 1\}$, with 0 (1) denoting the absence (presence) of the intervention. The Referee now challenges the players to produce outputs a and b, such that $a \oplus b = x \oplus y$. We define Alice's and Bob's *winning probability*,

$$P_{\text{win}} = \sum_{\substack{x,y=0 \\ a\oplus b = x\oplus y}}^{1} \frac{1}{4}\, p(ab|xy), \tag{1}$$

for this XOR game, where we have assumed the Referee's choice of interventions (x, y) to be uniformly distributed.

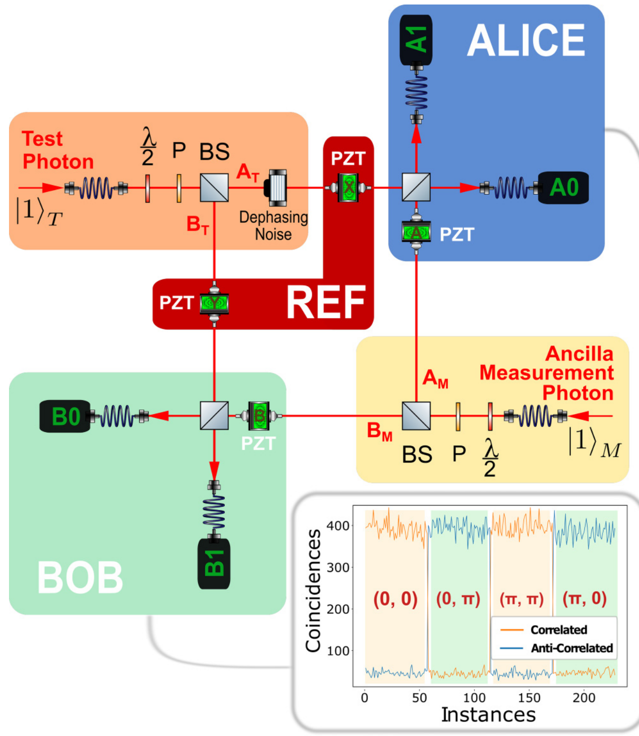

FIG. 1. XOR game implementation. The test (orange field) and ancilla measurement (yellow field) photons are generated via a spontaneous parametric down-conversion photon pair source (not pictured) and then coupled into the experiment, passing through a linear polarizer (P) to make them indistinguishable in polarization. Each photon is prepared in a coherent superposition of two spatial modes and sent to Alice (blue field) and Bob (green field). The Referee (red field) applies (or not) π phases to the spatial modes of the test particle through two piezo-enabled phase delays X, Y. Alice and Bob also each control a local phase (A, B), which they use to set their shared phase reference. They each locally interfere their test and ancilla measurement modes, recording coincidences between each other's detectors. Inset: Example data run. A plot of correlated (orange) and anticorrelated (blue) detection events as the Referee implements four different phase settings, delimited by shaded regions and indicated in parentheses. Switching the phase setting leads to a switch from correlated to anticorrelated detections. Each x value corresponds to one "instance" of the game, as described in the text.

For classical test particles, Ref. [26] showed that an optimum strategy employed by Alice and Bob will always yield $p(ab|xy) = 1/2$, and thus $P_{\text{win}} = 1/2$ which corresponds to random guessing. For a quantum superposition, however, Alice and Bob can find a strategy which yields $P_{\text{win}} > 1/2$. We will now show, that when the Referee's interventions are π-phase shifts instead of "path blockers" as originally imagined in Ref. [26], the quantum winning probability goes up to $P_{\text{win}} = 3/4$.

To determine the presence of the Referee's interventions, Alice and Bob use an ancillary measurement photon M, which is indistinguishable from T, apart from its spatial mode, and serves as a shared phase reference. Similarly to the test photon it is prepared in the superposition state $|\psi_M\rangle \equiv \frac{1}{\sqrt{2}}(|1,0\rangle_{A_M,B_M} + |0,1\rangle_{A_M,B_M})$, where A_M and B_M represent the

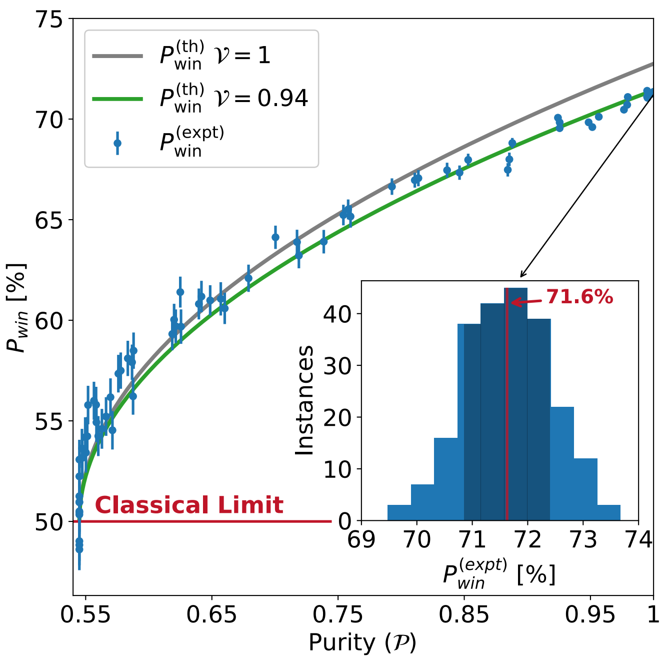

FIG. 2. Probability to win XOR game. As the superposition of the test particle is decohered, the winning probability $P_{\rm win}(\mathcal{P})$ approaches the classical limit. Blue dots are experimental data, and the error bars indicate the standard deviation taken over all instances. The average experimental win rate with a pure state is $P_{\rm win}^{(\rm expt)} = 0.716 \pm 0.007$, well above the classical limit. The gray curve shows Eq. (6) for perfect visibility, while the green curve corresponds to the experimentally measured HOM visibility $\mathcal{V} = 94\%$. Good agreement between the model and the data can be seen while the lower bound of 0.54 on the purity is due to imbalance in the preparation beam splitters. Inset: Distribution of experimental win rates for a single experimental run with a pure state. Each instance contains around 500 played games.

spatial modes of the measurement photon that are sent to Alice and Bob, respectively. In Fig. 2, the ancilla measurement photon is prepared at a central location and shared between their laboratories. In general, Alice or Bob could also prepare the photon in their laboratory and send the second mode to the other laboratory. They then perform joint measurements on the two photons consisting of simple coincidence detections, in contrast to past single-photon Bell violation experiments, which require complex homodyne measurements [22–25].

The state of the joint test-measurement system after the initial beam splitters is $|\psi_T\rangle|\psi_M\rangle$, where $|\psi_T\rangle$ and $|\psi_M\rangle$, defined above, are states over the test and measurement photon's modes A_T, B_T and A_M, B_M, respectively. As the two photons travel from the beam splitters to the measurement setups of Alice and Bob, the terms corresponding to each spatial mode acquire relative phases. We will denote the phases applied by the Referee by φ_x and φ_y, and set the propagation phases for the ancilla measurement photon to zero for simplicity (see Supplemental Material for a discussion [36]). The ideal premeasurement state is therefore

$$\frac{1}{\sqrt{2}}(e^{i\varphi_x}|1,0\rangle_{A_T,B_T} + e^{i\varphi_y}|0,1\rangle_{A_T,B_T}) \otimes |\psi_M\rangle. \quad (2)$$

We will use the ancillary measurement photon in the state $|\psi_M\rangle$ as a resource to verify the superposition between modes A_T and B_T. We therefore assume that the state $|\psi_M\rangle$ is free from error. Note that if $|\psi_M\rangle$ is imperfect, the success of our protocol will decrease. To perform their measurements, each party interferes their test and ancilla measurement modes on a 50/50 beam splitter, and detects which port the photons exit from (blue and green fields in Fig. 2). Note that before these beam splitters, we can consider the test photon to be distinct from the measurement photon since the modes that the test photon is superposed in are orthogonal to those of the measurement photon. However, since the test and measurement photons are indistinguishable (apart from their spatial modes), after mixing these modes on beam splitters one can no longer consider separate test and measurement photons.

Within each laboratory, Alice and Bob use their detection events to determine the Referee's action. Notice that half of the time both photons will arrive either in Alice's or Bob's laboratory. In this case the detection events contain no joint phase information and, again, the best the parties can do is to randomly guess the value of $x \oplus y$. The other half of the time, both parties receive one photon each. In this case, although there is no single-photon interference [Eq. (A2) in the Supplemental Material [36]] the probabilities for Alice and Bob's two-photon detection events to be correlated or anticorrelated are complementary and depend on $\varphi_x + \varphi_y$ [Eqs. (A3) and (A4) in the Supplemental Material [36]]. This occurs even though the modes these phases are applied on are not interfered. In other words, the detection events depend on delocalized combinations of the individual phases.

In order to phrase this scenario as an XOR game, we restrict the Referee's phases to $\varphi_x, \varphi_y \in \{0, \pi\}$, and write $\varphi_x = x\pi$, $\varphi_y = y\pi$. It follows that the anticorrelated events vanish if the Referee's choices satisfy $x \oplus y = 0$. Similarly, if the Referee chooses bits such that $x \oplus y = 1$ the correlated events vanish. Thus, when Alice and Bob both register a photon, they can win the game by simply outputting the index of the detector that registered a click. In the general case, the probability for Alice and Bob to give outputs a and b given Referee choices x and y is

$$p(ab|xy) = \frac{1}{4}\left[\frac{1}{2} + \cos^2\left(\frac{(x+y)\pi + (-1)^{a\oplus b}\pi}{2}\right)\right]. \quad (3)$$

Averaging this expression over all settings x and y yields a probability to win the game of $P_{\rm win} = 3/4$ when Alice and Bob output $a \oplus b$. However, this expression only holds true for perfectly indistinguishable particles in pure quantum states.

Experimental details. We generate the test and ancilla measurement photons using spontaneous parametric down-conversion (SPDC) in a type-II beta barium borate (BBO) crystal (see Supplemental Material [36]). To implement the interventions, the Referee is given control over two free-space delay stages, which are controlled by piezoelectric transducers (PZTs). Alice and Bob's local measurements are each implemented with a 50/50 beam splitter and a pair of single-photon detectors (A_0, A_1 and B_0, B_1, respectively).[1] Using two

[1]The phase calibration and detector efficiency measurements are described in detail in the Supplemental Material [36].

photons from an SPDC event ensures high-visibility two-photon interference. Nevertheless, imperfections remain. In the Supplemental Material [36], we compute P_{win} in the presence of our main experimental imperfections for a test particle described by the density matrix

$$\rho_T = \begin{bmatrix} \mathcal{T}_T & \lambda t_T r_T^* \\ \lambda t_T^* r_T & \mathcal{R}_T \end{bmatrix}, \tag{4}$$

where λ represents the amount of decoherence. The measurement particle state ρ_M is in the analogous pure state with $\lambda = 1$. Here, $\mathcal{T}_i = |t_i|^2$ and $\mathcal{R}_i = |r_i|^2$ describe the beam splitters used to superpose the photons $i \in \{T, M\}$. The main imperfections are the Hong-Ou-Mandel (HOM) visibility $\mathcal{V}$ between the test and ancilla measurement photons and the imbalance of the two input beam splitters, which further reduce P_{win}. The expression for the winning probability accounting for these factors is

$$P_{\mathrm{win}}(\lambda) = \tfrac{1}{2} + \tfrac{1}{2}\lambda\mathcal{V}(\mathcal{T}_T\mathcal{R}_M + \mathcal{T}_M\mathcal{R}_T). \tag{5}$$

To estimate the expected experimental win rates, we measure the HOM visibility on both detection beamsplitters (shown in Fig. 4 of the Supplemental Material [36]), finding a visibility of $\mathcal{V} = 94 \pm 2\%$. We also measure the splitting ratio of all beam splitters, finding that both input beam splitters have the same $\mathcal{R} : \mathcal{T}$ ratio of 0.65 : 0.35, while the detector beam splitters in Alice's and Bob's laboratories are balanced within experimental uncertainty. This simplifies Eq. (5) to

$$P_{\mathrm{win}}(\mathcal{P}) = \frac{1}{2} + \mathcal{V}\sqrt{\frac{\mathcal{R}\mathcal{T}}{2}}\sqrt{\mathcal{P} - (\mathcal{R}^2 + \mathcal{T}^2)}, \tag{6}$$

where we have now replaced λ with its expression for purity $\mathcal{P}$ from Eq. (C5). Setting $\mathcal{P} = 1$ gives an expected maximum winning probability of $P_{\mathrm{win}} = 0.7162$.

Results. Each experimental run consists of 240 instances of the game, with 60 instances for each phase setting (φ_x, φ_y). For each instance, we acquire coincidence counts for 1 s, yielding approximately 500 coincidence counts per second, distributed across all four coincidence patterns. Each coincidence count corresponds to one round of the XOR game. One experimental run thus amounts to approximately 120 000 rounds of the XOR game. The data in the inset of Fig. 2 constitute one experimental run, where the shaded areas indicate the two XOR sum values. To avoid bias, the order of the four phase settings is determined randomly for each run. The analyzed results of one run are displayed in the inset of Fig. 3. Therein we see the distribution of the experimental P_{win} over the 240 instances. For these data, an average win rate of 0.716 ± 0.007 is achieved, which is far above the classical limit of 0.50, and matches the maximum expected P_{win} given by Eq. (6). The PYTHON analysis can be found here [37]. Thus we can directly conclude that the test photon is in a coherent superposition.

To study the transition from the quantum regime to the classical limit, we decohere the test photon's spatial superposition by introducing controlled randomness in the test photon phase. We do so by adding phase noise with a Gaussian distribution to the Referee's X PZT setting for each instance. The standard deviation of the Gaussian distribution determines the amount of decoherence λ and thus the purity of the test photon. As described in the Supplemental Material [36], we can tune the purity of the test photon in the range [0.54,1], where the lower bound is due to the slight imbalance in the spatial superposition state. We then implement measurement runs, as defined above, for a set of purities in this range. The resulting win rates are plotted in the main panel of Fig. 3. As we vary the purity from 1 to 0.54, the win rate decreases according to the predicted experimental $P_{\mathrm{win}}(\mathcal{P})$ from Eq. (6). This prediction, plotted in green in Fig. 3, agrees well with our experiment, without using any free parameters. This measurement set further confirms the utility of XOR games for coherence detection, as even low-purity, almost classical superpositions can be effectively verified without the need to reinterfere the spatial modes.

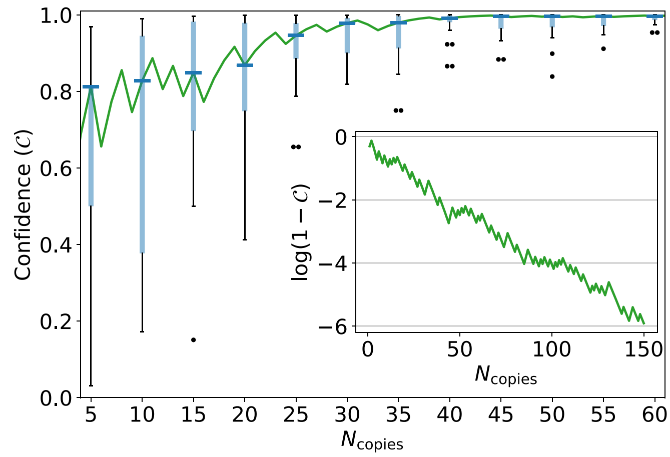

FIG. 3. Efficient confidence estimation. The median confidence, taken over 25 repetitions of the game, vs the number of rounds is indicated by the green line. The box plot illustrates the width of the confidence distributions, and the black dots show the number of outliers at the corresponding confidence value. As shown in the inset, the residual of the median confidence approaches zero exponentially fast in the number of detection events, and 37 copies suffice to reach a confidence above 99%.

By building on works exploring efficient verification of entanglement [27,28], our formulation of the verification task as an XOR game also allows us to verify superpositions efficiently. More concretely, we can express the confidence $\mathcal{C}$ that the test particle is in a superposition as $\mathcal{C} = 1 - p$, where $p = 1 - \sum_{k=0}^{N_{\mathrm{win}}-1} \binom{N}{k} \frac{1}{2^N}$ is the p value for the state not being in a superposition. This p value corresponds to the probability of a classical particle having generated at least as many wins as experimentally observed [see Supplemental Material Eqs. (E1) and (E2) [36]]. The confidence can therefore be interpreted as the probability of the particle having been in a superposition. We evaluate the median experimental confidence over 25 repetitions of the game, and find that in the majority of rounds 37 copies suffice to certify the superposition to 99 % confidence level (see Fig. 3). Moreover, as shown in the inset of the figure, the confidence approaches unity exponentially fast with the number of copies.

Discussion. In this Letter, we have demonstrated a protocol to detect the superposition of a quantum particle using spatially separated local measurements and two-photon interference with an identical resource photon. To do so, we created a delocalized two-photon interferometer, in which the individual phases applied to two modes of a spatially superposed photon are measured at a distance, without interfering these two modes with each other. While there is no single-photon interference in our work, it nevertheless relies on two-photon-like interference between indistinguishable photons. Our method to verify superposition can be contrasted with the indirect inference of spatial superposition through single-particle self-interference, such as in Young's double-slit experiment. Recent work has argued that such interference experiments admit classical explanations [38]. It would thus be interesting to analyze our nonlocal interferometer in light of this. The experimental apparatus we employ is similar to single-photon Bell tests or EPR steering experiments [24,25], with two crucial differences. First, the shared resource between the two parties in our work is a delocalized single-photon state, instead of a phase reference set with laser light. This allows us to stay in the discrete variable picture, and eliminates the need for complex measurements based on homodyne detection. Second, by designing an XOR game for the task of coherence detection we directly confirm superposition in a device-dependent, distributed framework providing a means to verify superposition, analogous to the use of entanglement witnesses, rather than Bell violations, to detect entanglement. Finally, to offset the additional resource cost of our protocol we use it in a shot-by-shot manner to achieve a confidence that the particle is superposed that converges to unity exponentially fast, providing an efficient tool to certify quantum superposition in quantum networks and distributed settings.

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