Superdiffusive Transport in Chaotic Quantum Systems with Nodal Interactions

Yu-Peng Wang,[1,2,3,*] Jie Ren,[1,4,*] Sarang Gopalakrishnan,[5] and Romain Vasseur[6]
[1]*Beijing National Laboratory for Condensed Matter Physics and Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*
[2]*University of Chinese Academy of Sciences, Beijing 100049, China*
[3]*Institute of Science and Technology Austria (ISTA), Am Campus 1, 3400 Klosterneuburg, Austria*
[4]*School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom*
[5]*Department of Electrical and Computer Engineering, Princeton University, Princeton, New Jersey 08544, USA*
[6]*Department of Theoretical Physics, University of Geneva, 24 quai Ernest-Ansermet, 1211 Genève, Switzerland*

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We introduce a class of interacting fermionic quantum models in d dimensions with nodal interactions that exhibit superdiffusive transport. We establish nonperturbatively that the nodal structure of the interactions gives rise to long-lived quasiparticle excitations that result in a diverging diffusion constant, even though the system is fully chaotic. Using a Boltzmann equation approach, we find that the charge mode acquires an anomalous dispersion relation at long wavelength $\omega(q) \sim q^z$ with dynamical exponent $z = \min[(2n+d)/2n, 2]$, where n is the order of the nodal point in momentum space. We verify our predictions in one-dimensional systems using tensor-network techniques.

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Introduction—The study of the emergence of hydrodynamic behavior and of the transport of conserved quantities such as charge, spin, or energy in many-body quantum systems has attracted significant interest in recent years [1–9]. Generic interacting lattice systems with short-range interactions exhibit diffusive transport at finite temperature, as expected from general hydrodynamic principles, see, e.g., Refs. [7,10–17], and finding generic deviations from diffusion is an important challenge. Slower than diffusive (subdiffusive) transport can naturally occur as a result of disorder and localization phenomena [13,18–23], or because of kinetic constraints [24–32]. On the other end of the spectrum, integrable systems support stable quasiparticle excitations and generically show ballistic transport.

Superdiffusive transport—between diffusive and ballistic—is, however, particularly elusive. In systems with long-range interactions, superdiffusion can naturally occur in the form of Lévy flights [6,33–38]. Momentum-conserving systems can also exhibit superdiffusion in one dimension [39–43]. However, in short-range lattice models, superdiffusion seems to require integrability: In noninteracting systems, superdiffusion has been shown to emerge for specific types of disorder [44–53] or dephasing [54,55]. In interacting systems, superdiffusion, along with Kardar-Parisi-Zhang–like scaling functions, has been observed in integrable models with non-Abelian symmetries [16,56–62]. This superdiffusive behavior appears to be remarkably stable to symmetry-preserving integrability-breaking perturbations [63–65], but the ultimate fate of transport is believed to be diffusive away from integrable points. To our knowledge, the only potential example of a nonintegrable lattice model exhibiting superdiffusion came from a recent numerical study [66] that indicates superdiffusive energy transport in the PXP and related models up to long timescales: a theoretical explanation for this phenomenon is still lacking.

In this Letter, we introduce a systematic method for constructing chaotic (nonintegrable) lattice models with superdiffusive charge transport. Motivated by the anomalous properties of noninteracting systems with a nodal structure [52,54], we construct "interacting," chaotic superdiffusive models from two key elements. The first ingredient is a free fermion Hamiltonian $\hat{H}_0 = \sum_k \epsilon(k)\hat{n}_k$, that supports stable quasiparticle excitations and ballistic transport. The second ingredient is a nodal interaction $\hat{V} = \sum_i \hat{V}_i$, where all local interaction terms $\hat{V}_i$ satisfy $[\hat{V}_i, \hat{n}_{k_0}] = 0$ for a specific momentum k_0. The interaction couples quasiparticles with different momenta, and generically leads to thermalization and chaotic behavior. However, due to the nodal structure $[\hat{V}_i, \hat{n}_{k_0}] = 0$, the quasiparticle lifetime diverges asymptotically near momentum k_0, leading to anomalous transport. For such interacting systems, $\hat{n}_k$ is approximately conserved as $k \to k_0$ up to very long times. Using a time-dependent Mazur-like bound [67], we obtain a divergent lower bound for the

[*]These authors contributed equally to this work.

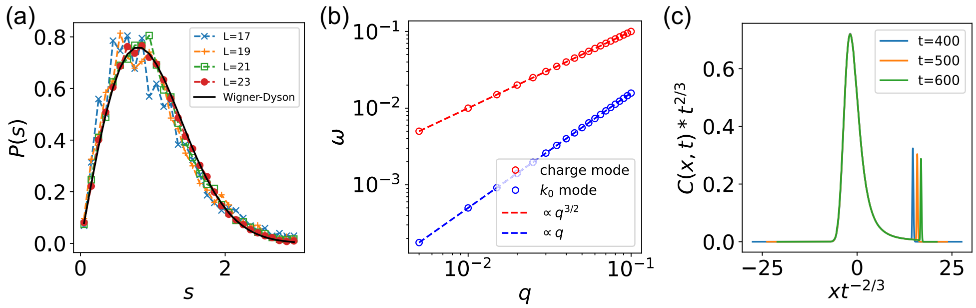

FIG. 1. Superdiffusion in chaotic nodal chains. (a) Distribution of many-body level spacing s in the middle half of the spectrum of nodal interaction model with $\tilde{\phi}_k = 1 + e^{i(k+\pi/2)}$. In this figure, we choose momentum $k = 0$, particle number $N = (L+1)/2$, and interaction strength $W = 4$. The r ratio [68] is consistent with Wigner-Dyson Gaussian orthogonal ensemble. There is clear level repulsion in this model, ruling out integrability. (b) Low-energy spectrum (real part) of normal modes of the linearized Boltzmann equation for nodal interactions with $\tilde{\phi}_k = 1 + e^{i(k+\pi/2)}$, exhibiting a node of order $n = 1$ at $k_0 = \pi/2$. We choose the interactions to be noisy in time $[\gamma(k) = \beta(k,k') = 1]$ to break energy conservation and focus on charge transport. We find two gapless modes: a ballistic mode corresponding to n_{k_0} (which is exactly conserved by the interactions), and a hydrodynamic charge mode with dispersion relation $\omega \sim q^{3/2}$ with q the momentum, indicating superdiffusive transport with dynamical exponent $z = 3/2$. (c) Structure factor $C(x,t) \equiv \langle n(x,t)n(0,0)\rangle$ in same setup as (b), obtained from the Boltzmann equation. The main component of the structure factor follows the scaling relation $C(x,t) = t^{-2/3} f(xt^{-2/3})$, consistent with the dynamical exponent $z = 3/2$. The decreasing ballistic component originates from modes near the node k_0.

diffusion constant, thereby establishing superdiffusion nonperturbatively. We also characterize transport using a Boltzmann equation approach, and find that the charge mode is governed by a universal dynamical exponent $z = (2n+1)/2n$, where n is the order of the node. We verify our results numerically using tensor-network techniques.

Models with nodal interactions—We consider fermionic systems in d dimensions subject to interactions with a nodal structure. More precisely, we construct local operators $\hat{V}_i$ that do not affect quasiparticles with a specific momentum k_0, i.e., $[\hat{V}_i, \hat{n}_{k_0}] = 0$, with $\hat{n}_k = \hat{c}_k^\dagger \hat{c}_k$. An important observation is that $[\sum_a \phi_a \hat{c}_{m+a}, \hat{n}_k] = e^{ikm} \hat{c}_k \sum_a \phi_a e^{ika}$, which implies that the operator $\hat{d}_m \equiv \sum_a \phi_a \hat{c}_{m+a}$ commutes with $\hat{n}_{k_0}$ for all sites m, provided the Fourier transform of its coefficient, $\tilde{\phi}_k \equiv \sum_a \phi_a e^{ika}$, has a node at k_0, i.e., $\tilde{\phi}_{k_0} = 0$. Building on this observation, we can construct interaction terms $\hat{V}_i$ from the operators $\hat{d}_i$s and $\hat{d}_i^\dagger$s, so modes with momenta k_0 are unaffected by this interaction. As we will demonstrate in this Letter, this structure is sufficient to enforce superdiffusive transport.

A similar nodal structure was employed to construct noninteracting noisy [54,55] and disordered systems [52,53] with anomalous properties. In this Letter, we generalize this construction to interacting systems, enabling the construction of chaotic (nonintegrable) superdiffusive systems. Our approach is very general and can be used in any dimension, but for the sake of simplicity we will focus on the following nodal one-dimensional spinless fermionic system:

$$H = \sum_k \epsilon(k) \hat{c}_k^\dagger \hat{c}_k + \sum_i W_i (\hat{d}_i^\dagger \hat{d}_i \hat{d}_{i+1}^\dagger \hat{d}_{i+1} + \text{H.c.}). \quad (1)$$

We will also set $W_i = W$, although we note that since the interaction strength W_i does not influence the nodal structure $[\hat{V}_i, \hat{n}_{k_0}] = 0$, our conclusions will also generalize to inhomogeneous interaction strength W_i. This model is generically nonintegrable (chaotic), and its level statistics follows a Wigner-Dyson distribution [Fig. 1(a)].

Before establishing superdiffusion in this model, we provide an intuitive understanding of its origin. Consider a system initially prepared as a Slater determinant of N quasiparticles with distinct momenta: $|k_1, \ldots, k_N\rangle$. When interactions are turned on, those states are obviously not eigenstates of the Hamiltonian anymore and thermalize. However, since $[\hat{n}_{k_0}, \hat{V}_i] = 0$, the lifetime of quasiparticles with momenta near k_0 diverges asymptotically. In the long-time limit, transport is dominated by the surviving quasiparticles, which carry charge ballistically. This hierarchy of long-lived quasiparticle excitations is responsible for superdiffusive behavior.

Divergent diffusion constant—To make this argument more precise, we first establish the divergence of the diffusion constant, corresponding to superdiffusive transport. The diffusion constant at infinite temperature can be calculated from the current-current correlation function from the Kubo formula as $D = 4\lim_{t\to\infty}\lim_{L\to\infty}(1/L)\int_0^t d\tau \langle \hat{J}(\tau)\hat{J}(0)\rangle$ [16,69,70], where $\langle \cdots \rangle = 2^{-L}\text{Tr}(\cdots)$, and $\hat{J}(t)$ is a sum of local currents over all lattice sites $\hat{J}(t) = \sum_r \hat{j}_r(t)$. We aim to show that the diffusion constant diverges in fermionic systems with nodal interactions.

The main idea is that since n_k is conserved at the node k_0, we can treat n_k as approximately conserved for momenta in the neighborhood $\mathcal{K} = (k_0 - \delta k, k_0 + \delta k)$, where δk will be time-dependent. To determine the size of the subset $\mathcal{K}$, we use the following bound [71] on the infinite-temperature autocorrelation of an arbitrary operator,

$$\langle(\hat{O}(t)-\hat{O}(0))\hat{O}(0)\rangle \leq \langle[\hat{H},\hat{O}][\hat{H},\hat{O}]^\dagger\rangle t^2/2. \quad (2)$$

We apply this bound (2) to the operator $\hat{n}_k$. We show by explicit calculation that $\langle[\hat{H},\hat{n}_k][\hat{H},\hat{n}_k]^\dagger\rangle \leq c^2|\tilde{\phi}_k|^2$ for some $O(1)$ constant c [72]. We consider $\hat{n}_k$ to be approximately conserved during this time interval $(0,t)$ if the right-hand side of Eq. (2) is small throughout the interval. Given a node of order n, i.e., $\tilde{\phi}_k \sim |k-k_0|^n$, for arbitrarily large t, as long as momenta are chosen such that $|k-k_0|^n t < O(\sqrt{\epsilon})$, n_k is approximately conserved up to accuracy ϵ. This leads to the scaling $\delta k(t) \sim t^{-1/n}$: for an arbitrary long time t, occupation numbers with momenta within $\delta k(t)$ of a node are approximately conserved.

We can then decompose the current into a slow component, $\hat{J}_s$, that remains conserved up to time t, and a fast component, $\hat{J}_f(t) = \hat{J}(t) - \hat{J}_s$, that decays quickly and contributes to a finite conductivity. The slow part is given by the (hydrodynamic) projection of the current onto the occupation numbers $\hat{n}_k$ with $k \in \mathcal{K}(t) = [k_0 - \delta k(t), k_0 + \delta k(t)]$. We have $\hat{J}_s = \sum_{k\in\mathcal{K}(t)} g(k)\hat{n}_k$, where the coefficients $g(k)$ are the overlaps of the current onto the occupation numbers—for weak interactions, we have $g(k) = v(k) + O(W)$ with $v(k) = \partial_k\epsilon(k)$. By definition, the slow part of the current is conserved up to times t: $J_s(\tau) \approx J_s(0)$ for $\tau \leq t$ (up to accuracy ϵ). Since the fast and slow components are orthogonal, we can derive a Mazur-like lower bound for the diffusion constant [72]

$$D \geq 4\lim_{t\to\infty}\lim_{L\to\infty}\frac{1}{L}\int_0^t \mathrm{d}\tau\langle\hat{J}_s^2\rangle \equiv D_s. \quad (3)$$

Our goal is to then show that this lower bound diverges. Using the explicit form of the slow part of the current, we have $\lim_{L\to\infty}(1/L)\langle\hat{J}_s^2\rangle = \int_{k\in\mathcal{K}(t)}(dk/2\pi)g(k)^2 n_k(1-n_k)$, which plays the role of a time-dependent Drude weight (note that this quantity does not depend on τ). For sufficiently long time t, $\delta k(t)$ becomes very small, so we have $\lim_{L\to\infty}(1/L)\langle\hat{J}_s^2\rangle \underset{t\to\infty}{\sim} \delta k(t)(1/2\pi)g(k_0)^2 n_{k_0}(1-n_{k_0})$. This gives

$$D_s \sim g(k_0)^2\lim_{t\to\infty} t\delta k(t), \quad (4)$$

with $\delta k(t) \sim t^{-1/n}$. This shows that the lower bound D_s diverges for any $n > 1$ as long as the overlap of the current with the node occupation does not vanish: $g(k_0) = v(k_0) \neq 0$ [see Boltzmann approach below, this is satisfied unless the velocity $v(k_0)$ vanishes at the node]. Therefore, the diffusion constant diverges in the nodal interaction model, and transport must be superdiffusive, at least for $n > 1$.

Boltzmann equation—Although the argument above establishes superdiffusion in our model in a nonperturbative way, it does not predict the dynamical exponent z associated with charge transport. More precisely, the argument above can be turned into a lower bound for an effective time-dependent diffusion constant $t^{1-1/n} \lesssim D(t)$, but the actual diffusion constant diverges faster with time (corresponding to a smaller exponent z): the $O(\tilde{\phi}_k)$ corrections to the commutator add up incoherently, not constructively. To estimate the true transport exponents we turn to a Boltzmann equation approach. First, we write the Hamiltonian [Eq. (1)] in momentum space $H = \int(\mathrm{d}k/2\pi)\epsilon(k)\hat{c}_k^\dagger\hat{c}_k + \int[\mathrm{d}k^4/(2\pi)^3]\delta_{2\pi}(\underline{k})U_{k_1,k_2,k_3,k_4}\hat{c}_{k_1}^\dagger\hat{c}_{k_3}\hat{c}_{k_2}^\dagger\hat{c}_{k_4}$, where $U_{k_1,k_2,k_3,k_4} = W\tilde{\phi}_{k_1}^*\tilde{\phi}_{k_2}^*\tilde{\phi}_{k_3}\tilde{\phi}_{k_4}\cos(k_2-k_4)$ and $\delta_{2\pi}(\underline{k}) = \delta(k_1+k_2-k_3-k_4 \bmod 2\pi)$. The corresponding Boltzmann equation has the general form $\partial_t n(x,k,t) + v(k)\partial_x n(x,k,t) = f_k^{\mathrm{col}}[n]$, where $n(x,k,t)$ represents the coarse-grained particle density at position x with momentum k, $v(k) = \partial_k\epsilon(k)$ and $f_k^{\mathrm{col}}[n(x,k,t)]$ denotes the collision integral term due to the interactions.

Crucially, the nodal structure $\partial_t\hat{n}_{k_0} = 0$ implies that the collision integral $f_{k_0}^{\mathrm{col}} = 0$. In turn, the nodal form of the interactions gives rise to diverging lifetimes for modes with momenta near k_0. Linearizing the Boltzmann equation around a thermal equilibrium state, and expanding the distribution function as $n(x,k,t) = n_{\mathrm{eq}} + \delta n(x,k,t)$, the general form of the Boltzmann equation for m-particle scattering is

$$f_k^{\mathrm{col}}[\delta n] = -\gamma(k)|\tilde{\phi}_k|^2\delta n(x,k,t) + \gamma(k)|\tilde{\phi}_k|^2\int \mathrm{d}k'\,\beta(k,k')\delta n(x,k',t), \quad (5)$$

where $\gamma(k)$, $\beta(k,k')$ are non-negative, and $\int \mathrm{d}k'\beta(k,k') = 1$. The specific form of $\gamma(k)$ and $\beta(k,k')$ are unimportant. The crucial point is that the matrix element involving momentum k for the scattering processes in the nodal problem all go as $\sim|\tilde{\phi}_k|$. Therefore, the collision integral (decay rate) goes as $\sim|\tilde{\phi}_k|^2$. Examples of specific collision integrals are discussed in Supplemental Material [72].

Long-lived normal modes—The general structure of the linearized Boltzmann equation is similar to the case of free fermions with dephasing noise with a nodal structure, studied in Refs. [54,73]. The linearized Boltzmann equation has a natural interpretation as a Markov process in momentum space: when $\tilde{\phi}_k \neq 0$ for all momenta k, the mean-free path $l_k = v(k)/[\gamma(k)|\tilde{\phi}_k|^2]$ is finite for all momentum k. In this case, the charge transport is diffusive. However, when $\tilde{\phi}_{k_0+q} \sim q^n$ for certain nodes k_0, the mean-free path diverges near these nodes, resulting in a fat-tailed distribution for the mean-free path $p(\ell) \sim \ell^{-2-1/(2n)}$.

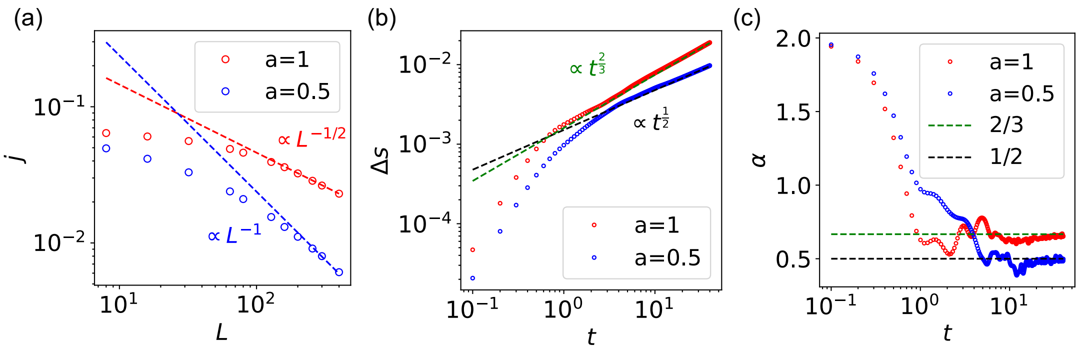

FIG. 2. Tensor-network simulations. (a) NESS current under boundary driving. When the Hamiltonian possesses node ($a = 1$), the scaling of NESS current and system size satisfies $j \sim L^{-1/2}$ at large L limit. In the case $a = 0.5$, the interactions do not exhibit a node, and the scaling of NESS current and system size satisfies $j \sim L^{-1}$ at large L limit consistent with diffusive transport. (b) Time dependence of the charge transfer Δs starting from a domain wall initial state. (c) The time-dependent exponent $\alpha(t)$ calculated as the numerical logarithmic derivative $\mathrm{d}\log\Delta s/\mathrm{d}\log t$. (b) and (c) demonstrate that in the model with nodal interaction, $\Delta s \propto t^{2/3}$, corresponding to a dynamical exponent $z = 3/2$. In contrast, with non-nodal interaction, $\Delta s \propto t^{1/2}$, indicating a dynamical exponent $z = 2$. For the NESS current simulations (a), we chose $\Gamma = 1$, $\mu = 0.02$, and a maximum bond dimension $\chi = 180$. In the charge transfer simulations (b) and (c), we chose $\nu = 0.01$ and a maximum bond dimension $\chi = 200$.

This is the hallmark of a Lévy walk, so charge transport becomes superdiffusive, with the corresponding dynamical exponent given by $z = (1 + 2n)/2n$. This anomalous scaling can be observed directly in the spectrum of normal modes of the linearized Boltzmann equation. In addition to the exactly ballistic conserved modes n_{k_0}, we find a normal mode corresponding to $\sum_{k\neq k_0} n_k$ with dispersion relation

$$\omega(q) \sim q^{(1+2n)/2n}, \tag{6}$$

see Figs. 1(b) and 1(c) for the corresponding dynamical structure factor. Although we focus on charge transport, energy transport is also superdiffusive following the same reasoning.

This argument can be easily generalized to higher dimensions. For the nodal interaction model in higher dimensions, the Hamiltonian is given by $H = \sum_{\boldsymbol{k}} \epsilon(\boldsymbol{k})\hat{c}^\dagger_{\boldsymbol{k}}\hat{c}_{\boldsymbol{k}} + W\sum_{\langle i,j\rangle} \hat{d}^\dagger_i\hat{d}_i\hat{d}^\dagger_j\hat{d}_j$. An example can be found in Supplemental Material [72]. The corresponding Boltzmann equation is analogous to the one-dimensional case, with the one-dimensional variables v, ∂_x, x, and k replaced by their higher-dimensional counterparts: the velocity vector $\boldsymbol{v} = \nabla_{\boldsymbol{k}}\epsilon(k)$, the gradient $\nabla_{\boldsymbol{r}}$, the position vector $\boldsymbol{r}$, and the momentum vector $\boldsymbol{k}$. In d dimensions, assuming the velocity at the nodes is nonzero, an m-dimensional nodal surface of order n yields a dynamical exponent $z = (2n + d - m)/2n$, which only holds true for $z < 2$. For example, nodal lines of order n in two-dimensional systems results in $z = (2n + 1)/2n$, and nodal points of order n in two-dimensional systems results in $z=(n+1)/n$ [54].

Numerics—To check our results numerically for one-dimensional fermionic chains, we use tensor-network (matrix-product operator) techniques [74]. We first consider a system with boundary driving, and analyze the scaling of the current in the nonequilibrium steady state (NESS), a method frequently employed to study transport properties [56,75–77]. In this setup, the first and last sites of the system are coupled to external baths, modeled phenomenologically by four Lindblad operators,

$$\begin{aligned} L_1 &= \sqrt{\Gamma(1+\mu)}\hat{c}^\dagger_1, \qquad & L_2 &= \sqrt{\Gamma(1-\mu)}\hat{c}_1, \\ L_3 &= \sqrt{\Gamma(1-\mu)}\hat{c}^\dagger_L, \qquad & L_4 &= \sqrt{\Gamma(1+\mu)}\hat{c}_L, \end{aligned} \tag{7}$$

where ρ is density matrix and $\mathcal{L}^{(\mathrm{bath})}(\rho) = \sum_{k=1}^4 2L_k\rho L_k^\dagger - \rho L_k^\dagger L_k - L_k^\dagger L_k\rho$. The evolution of the density matrix is then governed by the Lindblad master equation $(\mathrm{d}\rho/\mathrm{d}t) = \mathrm{i}[\rho, \hat{H}] + \mathcal{L}^{(\mathrm{bath})}(\rho)$. The NESS current scales with system size as $j \sim L^{-(z-1)}$; see e.g., Refs. [75,77].

We employ the time-evolving block decimation algorithm [76,78–80] to obtain the NESS of the Lindblad master equation [72]. The interaction strength in the Hamiltonian [Eq. (1)] is set to a uniform value, $W_i = 4$. For simplicity in simulations, we choose $\epsilon(k) = -\cos k$, $\hat{d}_i = (\hat{c}_i + ia\hat{c}_{i+1})/\sqrt{1+|a|^2}$, and correspondingly $\tilde{\phi}_k = 1 + ae^{i(k+\pi/2)}$. When $a = 1$, $\tilde{\phi}_k$ exhibits a node at $k_0 = \pi/2$ with order $n = 1$. We present numerical results for this model [Fig. 2(a)]. In the large L limit, the scaling of the NESS current with system size follows $j \sim L^{-1/2}$ at large L limit, indicating superdiffusive transport with a dynamical exponent $z = 3/2$, in agreement with the predictions of the Boltzmann equation. In contrast, when $a = 0.5$, $\tilde{\phi}_k \neq 0$ for all k, and the scaling of the NESS current satisfies $j \sim L^{-1}$ in the large L limit, corresponding to diffusive transport. In our simulation, the coupling strength Γ is set to $\Gamma = 1$ and the driving parameter μ is set to be small $\mu = 0.02$ to ensure we are in a linear response regime where all observables relevant for magnetization transport are proportional to μ.

To complement these results, we also use the density matrix truncation method [81] to study transport after a quantum quench from a domain wall initial state. The initial state is a domain wall state given by

$$\rho(t=0) \sim (1+\nu\sigma^z)^{\otimes\frac{L}{2}} \otimes (1-\nu\sigma^z)^{\otimes\frac{L}{2}}, \qquad (8)$$

where $\hat{\sigma}^z = 2\hat{n} - 1$ is a Pauli matrix in the spin language, using a standard Jordan-Wigner transformation. We investigate the cumulative charge flow (charge transfer) from left half to right half $\Delta s(t) \equiv \int_0^t j[x=(L/2), t']\mathrm{d}t'$. In the long-time limit, the asymptotic scaling of the cumulative charge Δs is governed by the dynamical exponent, $\Delta s(t) \sim t^{1/z}$; see e.g., Ref. [57]. To illustrate the generality of our results and to achieve better convergence of the dynamical exponent, we choose the interaction strengths W_i to be random, sampled uniformly from $[-4, 4]$. Again, we use small $\nu = 0.01$ to ensure we are in a linear response regime. The numerical results are presented in Figs. 2(b) and 2(c). For the nodal interaction case ($a = 1$), we find $\Delta s \propto t^{2/3}$, corresponding to a dynamical exponent $z = 3/2$. Conversely, in the non-nodal interaction case ($a = 0.5$), $\Delta s \propto t^{1/2}$, indicating a dynamical exponent $z = 2$.

Discussion—In this Letter, we constructed interacting, chaotic superdiffusive models using two key elements: (1) a free-fermion Hamiltonian that possesses ballistic eigenmodes and (2) nodal interactions, where each local term commutes with the particle number operator $\hat{n}_{k_0}$ with a specific momentum k_0. These two elements lead to a asymptotically divergent lifetime for quasiparticles near momentum k_0. Quasiparticles with momenta near k_0 are responsible for the superdiffusive transport. The dynamical exponent z is determined by the order n of the node, and is given by $z = (2n+1)/2n$ in one dimension.

Our Letter provides a counterexample to the conventional wisdom that chaotic lattice models exhibit diffusive behavior, offering a clear analytic explanation for the origin of superdiffusion. Although superdiffusion in these models relies on some degree of fine-tuning, our construction remains very general, and could provide an explanation for the observed superdiffusion in the PXP and related models, where isolated ballistic modes were also observed [66]. We established superdiffusion in a nonperturbative way, and the interaction strength does not affect the dynamical exponent. Superdiffusion can even occur for inhomogeneous (random) interaction strengths. Furthermore, our construction is not limited to one dimension—it is also possible to construct superdiffusive models in two and three dimensions.

It would be very interesting to extend our results to perturbed interacting integrable systems. Integrable systems generically possess stable, ballistic quasiparticle excitations. A key challenge would be to identify local operators in such modes that commute with the quasiparticle number operator at specific momenta. Using these operators as integrability-breaking perturbations would naturally lead to superdiffusive behavior. We leave this extension for future work.

The numerical simulations based on tensor networks use the ITensor package [82].

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Data availability—The data that support the findings of this article are openly available [83].

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